Received: *October 13, 2001*, Accepted: *November 7, 2001*

Superstars and giant gravitons

Robert C. Myers

Perimeter Institute for Theoretical Physics
35 King Street North, Waterloo, Ontario N2J 2W9, Canada
Department of Physics, University of Waterloo
Waterloo, Ontario N2L 3G1, Canada
Department of Physics, McGill University
Montréal, Québec H3A 2T8, Canada
E-mail: rcm@hep.physics.mcgill.ca

Øyvind Tafjord

Wolfram Research, 100 Trade Center Drive
Champaign, Illinois 61820, USA
E-mail: tafjord@wolfram.com

Abstract: We examine a family of BPS solutions of ten-dimensional type-IIB supergravity. These solutions asymptotically approach $\mathrm{AdS}_5 \times S^5$ and carry internal 'angular' momentum on the five-sphere. While a naked singularity appears at the center of the anti-de Sitter space, we show that it has a natural physical interpretation in terms of a collection of giant gravitons. We calculate the distribution of giant gravitons from the dipole field induced in the Ramond-Ramond five-form, and show that these sources account for the entire internal momentum carried by the BPS solutions.

Keywords: D-branes, Black Holes in String Theory, AdS-CFT Correspondance.

JHEP11(2001)009

Contents

1. Introduction

Brane expansion is a fascinating effect which string theory seems to employ in a wide variety of circumstances to regulate divergences and resolve singularities [1, 2, 3, 4]. Giant gravitons [3] provide one such example where brane expansion is exploited in realizing the stringy exclusion principle [5]. Naively, type-IIB supergravity in an $\mathrm{AdS}_5 \times S^5$ background has infinite towers of BPS states associated with excitations of the spherical harmonics on the five-sphere [6]. Through the AdS/CFT correspondence [7, 8], these seem to be related to a family of chiral primary operators in the dual $\mathcal{N} = 4$ $\mathrm{U}(N)$ super-Yang-Mills theory. However, the latter family of operators is cutoff because the rank of the gauge group is finite. This cut-off is precisely reproduced by superstring theory in the $\mathrm{AdS}_5 \times S^5$ background by realizing these states as spherical D3-branes that expand on the background five-sphere with increasing angular momentum [3]. The cut-off arises because the expansion is limited by the finite volume of the five-sphere. Hence the apparently infinite towers of BPS states are truncated in the full string theory through the mechanism of brane expansion.

However, the precise dictionary between the dual AdS and CFT theories remains incomplete with regards to giant gravitons. Recently evidence was presented that the giant gravitons are not directly dual to single trace chiral primaries but rather they are dual to a family of subdeterminant operators, where the finite rank of the gauge group also imposes the same cut-off [9]. Further, one finds that there are 'dual' giant gravitons which expand into the AdS space and which experience no 'angular momentum' cut-off [10, 11, 12]. The test brane analysis also indicates that there is a zero-size configuration with the same mass and internal momentum [3, 10, 11].

So far, giant gravitons have only be analyzed as test branes propagating in a fixed background spacetime. Certainly the AdS/CFT dictionary would be clarified with a better understanding of higher quantum effects on the CFT side or gravitational back reaction on the AdS side. While the supersymmetric nature of these states will ensure that certain properties are not modified by such higher order corrections, we certainly expect that the detailed dynamics of the giant gravitons will change. As a step towards understanding gravitational back reaction, we will study a certain family of solutions of the full nonlinear equations of motion in type-IIB supergravity. These solutions asymptotically approach $AdS_5 \times S^5$ and are charged by carrying internal momentum on the S^5. Hence they have precisely the properties one expects for solutions describing giant graviton configurations.

To begin, we consider solutions within five-dimensional $\mathcal{N} = 2$ supergravity coupled to two abelian vector supermultiplets. This theory with a $U(1)^3$ gauge group can be embedded in the $\mathcal{N} = 8$ theory with SO(6) gauge symmetry obtained by reducing type-IIB supergravity on an S^5. The five-dimensional theory has a family of black hole solutions parameterized by three charges and a non-extremality parameter μ [13, 14]. The supersymmetric limit for these solutions corresponds to $\mu = 0$, however, at this point, the event horizon has disappeared and so these BPS solutions are *not* black holes. Rather they contain naked singularities, a feature that was first noted in ref. [15]. Given the absence of a horizon, we refer to the BPS solutions as 'superstars'.

Now in general, one can interpret the appearance of a naked singularity in a (super)gravity solution as indicating the presence of an external source. In many cases, the properties of source are such that the solution is clearly unphysical [16]. However, in certain cases, one finds that the source has a reasonable physical interpretation, especially in string theory where the full theory has many extended brane sources — see, e.g., [17]. The main result of the present paper is to give a physical interpretation for the source at the singularity of the BPS solutions described above.

To address this question properly, however, we must lift the solutions to the full ten-dimensional IIB supergravity, following refs. [18, 19, 20]. We find that the BPS solutions in ten dimensions inherit a naked singularity from their five-dimensional reductions. In this framework, we find that a strong dipole field is excited in the Ramond-Ramond (RR) five-form near the singularity. Thus the source appears to be a collection of spherical D3-branes, i.e., an ensemble of giant gravitons. From the details of the five-form field, we are able to derive the precise distribution of giant gravitons on the five-sphere, which acts as a source for the ten-dimensional superstars. Further we show that the entire internal momentum of the BPS solutions is accounted for by the giant gravitons.

An outline of the paper follows: We start in the next section by reviewing the charged AdS_5 black hole solution. In section 3, we then present our interpretation of the singular BPS solutions in terms of a distribution of giant gravitons. In section 4, we present calculations in which we probe the supergravity solutions with dual giant gravitons. Finally, we conclude with a discussion section.

2. Charged AdS black holes

To begin, we consider the S^5 reduction of type-IIB supergravity [6, 21], truncated to the $\mathcal{N} = 2$ sector with $\mathrm{U}(1)^3$ gauge symmetry. While constructing the full reduced theory is very complex — see, e.g., [22] — the latter five-dimensional theory has long been known [23] (see also, e.g., [20] for a review). The matter content includes two scalar fields and the three U(1) gauge fields A_i ($i = 1, 2, 3$). The scalar fields are conveniently parametrized in terms of three scalars X_i subject to the constraint $X_1X_2X_3 = 1$. This five-dimensional theory has following three-charge black hole solutions [13, 14],

$$ds_5^2 = -(H_1H_2H_3)^{-2/3} f dt^2 + (H_1H_2H_3)^{1/3}(f^{-1}dr^2 + r^2 d\Omega_3^2)\,, \tag{2.1}$$

$$X_i = H_i^{-1}(H_1H_2H_3)^{1/3}\,, \qquad A_i = (H_i^{-1} - 1)dt\,, \tag{2.2}$$

where

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L^2}H_1H_2H_3\,, \qquad H_i = 1 + \frac{q_i}{r^2}\,. \tag{2.3}$$

We choose the angular coordinates such that line-element on the three-sphere is given by

$$d\Omega_3^2 = d\alpha_1^2 + \sin^2\alpha_1\left(d\alpha_2^2 + \sin^2\alpha_2\, d\alpha_3^2\right). \tag{2.4}$$

Hence the horizons appearing in these solutions have a spherical topology. While there exist analogous solutions where the horizon has zero or constant negative curvature, we will not discuss these solutions here. The parameter μ is a 'non-extremality' parameter, where $\mu = 0$ corresponds to the supersymmetric BPS solution. The (positive) parameters q_i can be related to the physical U(1) charges $\tilde{q}_i$ by

$$\tilde{q}_i = \sqrt{q_i(\mu + q_i)}\,. \tag{2.5}$$

For the BPS solutions, with which we will mostly be concerned, we therefore have $\tilde{q}_i = q_i$ since $\mu = 0$. Finally the mass of this black hole is [14]

$$M = \frac{\pi}{4G_5}\left(\frac{3}{2}\mu + \sum q_i\right). \tag{2.6}$$

Note that in the special case $q_1 = q_2 = q_3$, the scalar fields in eq. (2.2) are trivial and the solution reduces to the five-dimensional Reissner-Nordstrøm-anti-de Sitter geometry [15]. The thermodynamic properties of this equal-charge solution [18] and the general case [24, 25] were extensively studied in context of the AdS/CFT correspondence [7, 8].

In the absence of any scalar field excitations, the lift of the equal-charge solution to ten dimensions is relatively straightforward [18]. The general solution (2.2) has also been lifted to the ten-dimensional IIB supergravity theory yielding the following geometry [19]

$$ds_{10}^2 = \sqrt{\Delta}\left[-(H_1H_2H_3)^{-1}f\,dt^2 + (f^{-1}dr^2 + r^2 d\Omega_3^2)\right] + \\ +\frac{1}{\sqrt{\Delta}}\sum_{i=1}^{3} H_i\left(L^2 d\mu_i^2 + \mu_i^2[L d\phi_i + (H_i^{-1}-1)dt]^2\right), \tag{2.7}$$

with

$$\Delta = H_1H_2H_3\sum_{i=1}^{3}\frac{\mu_i^2}{H_i}, \qquad \mu_1 = \cos\theta_1\,, \qquad \mu_2 = \sin\theta_1\cos\theta_2\,, \qquad \mu_3 = \sin\theta_1\sin\theta_2\,. \tag{2.8}$$

The appearance of off-diagonal terms in the above metric illustrate that the U(1) gauge charges correspond to internal momenta along the three Killing coordinates $\phi_{1,2,3}$ in the ten-dimensional setting. The full solution also contains a self-dual Ramond-Ramond (RR) five-form field, which may be written $F^{(5)} = dB^{(4)} + *dB^{(4)}$ where

$$B^{(4)} = -\frac{r^4}{L}\Delta\,dt\wedge d^3\Omega - L\sum_{i=1}^{3} q_i\mu_i^2(L\,d\phi_i - dt)\wedge d^3\Omega \tag{2.9}$$

Above $d^3\Omega \equiv \sin^2\alpha_1\sin\alpha_2\,d^3\alpha$ denotes the volume element on the unit three-sphere. In the limit where all the charges q_i vanish, we recover the maximally supersymmetric $\mathrm{AdS}_5\times S^5$ background with radius of curvature L.

We will in the following focus on the ten-dimensional BPS solution with $\mu = 0$. First we note that it is only for μ greater than some critical value that the above solutions are true black holes with a regular horizon [14]. As a result, the BPS solutions all contain naked singularities [15] and hence do not correspond to extremal black holes, i.e., with an event horizon with vanishing surface gravity. This remains true whether the BPS solutions are interpreted in terms of supergravity in ten or five dimensions. Again, given the absence of an event horizon, we refer the BPS solutions as 'superstars'. We will briefly consider the non-BPS configurations in the discussion section.

The naked singularities will be central in the following and so we discuss some of their properties here. First note that the detailed nature of the singularity changes depending on whether only a single charge or more than one of the charges are non-vanishing. A common characteristic shared by all cases is that while the singularity is located at the center of the AdS_5 space, it also extends over the entire S^5 with varying strength, i.e., the rate at which curvature invariants diverge as $r\to 0$ varies with the angles $\theta_{1,2}$.

To discuss the differences and throughout the rest of the paper, we will assume without loss of generality that

$$q_1 \geq q_2 \geq q_3 \tag{2.10}$$

If only q_1 is nonzero, the nonextremal horizon shrinks to zero size as μ approaches zero. At precisely $\mu = 0$, the horizon disappears and $r = 0$ becomes a null singularity. If q_1 and q_2 are nonzero but $q_3 = 0$, the nonextremal horizon shrinks to zero size as $\mu \to \mu_{crit} = q_1 q_2 / L^2$. At precisely $\mu = \mu_{crit}$, $r = 0$ becomes a null singularity. For $0 \leq \mu < \mu_{crit}$ (and hence for the BPS solution), $r = 0$ is a time-like singularity. Finally if all three charges are nonzero, the horizon again vanishes for μ smaller than some finite critical value. The precise value of μ_{crit} may be determined analytically [14], but here we simply observe that $\mu_{crit} > (q_1 q_2 + q_1 q_3 + q_2 q_3)/L^2$. For this solution, one can continue the spacetime geometry past $r = 0$. Defining a new radial coordinate

$$\rho^2 = r^2 + q_3 \,, \tag{2.11}$$

$\rho \geq 0$ covers the whole geometry and the naked singularity appears at $\rho = 0$ [14]. For $\mu < \mu_{crit}$, this singularity is again timelike, and extends over the entire S^5 with varying strength for generic values of the charges.

3. Superstar Goliath

One can interpret the appearance of a naked singularity in a supergravity solution as indicating the presence of an external source. Hence one should not necessarily rule out such solutions as unphysical, but rather ask if it has a reasonable physical source [17]. Here we will argue that the source appearing in the superstar solutions has a natural interpretation within string theory as an ensemble of giant gravitons distributed over the S^5 and located at the origin of AdS_5 geometry.

In the present context [3], a giant graviton is a spherical D3-brane with fixed internal 'angular momentum' around a circle of the S^5. Studying the equations of motion of a test D3-brane in this configuration, one finds that it expands into a finite sized three-sphere within the S^5. Given the coordinate parameterization in eqs. (2.7) and (2.8), it is convenient to consider a giant graviton[1] moving along ϕ_1. In this case, the expansion occurs in the θ_1 direction and the resulting three-sphere is parameterized by θ_2, ϕ_2, ϕ_3 with a (proper) area of $2\pi^2 L^3 \sin^3\theta_1$. The calculations of ref. [3] show that for angular momentum P_{ϕ_1}, the size of the giant graviton is given by

$$\sin^2\theta_1 = \frac{P_{\phi_1}}{N} \,, \tag{3.1}$$

[1] We are only considering here the motion of a test D3-brane in the $\mathrm{AdS}_5 \times S^5$ background, i.e., in the solution with $q_i = 0$ and $\mu = 0$. In the next section, we present an analysis of the motion of certain test branes in the full superstar background.

while the energy is

$$E = \frac{P_{\phi_1}}{L} \tag{3.2}$$

in agreement with the BPS bound. In eq. (3.1), N indicates the number of flux quanta for the RR five-form in the background. This integer is related to the radius of curvature of the AdS$_5$ and S^5 geometries by [8]

$$L^4 = 4\pi g_s N \ell_s^4 \tag{3.3}$$

and so is implicitly understood to be large (and positive). Some other useful relations in the following are for Newton's constant (in five and ten dimensions) and the D3-brane tension:

$$G_5 = \frac{G_{10}}{(\pi^3 L^5)}\,, \qquad G_{10} = 8\pi^6 g_s^2 \ell_s^8\,, \qquad T_3 = \frac{1}{(8\pi^3 g_s \ell_s^4)}\,. \tag{3.4}$$

In any event, as $\sin^2\theta_1 \leq 1$, eq. (3.1) yields precisely the desired angular momentum bound: $P_{\phi_1} \leq N$.

Now we turn to the physical interpretation of the naked singularity appearing in the superstar. The source is conjectured to be a collection of giant gravitons. These spherical branes correspond to D3-brane dipoles. Hence while these spheres carry no net D3-brane charge, they will locally excite the RR five-form field. However if we consider a small (five-dimensional) surface which encloses a portion of the sphere, this surface will carry a net five-form flux proportional to the number of D3-branes enclosed. That is, we must choose a surface that is the boundary of a six-dimensional 'ball' which only intersects the three-sphere of the giant graviton once at a point. Hence if the above conjecture is correct, by considering an appropriate infinitesimal near the superstar singularity, we should find a net five-form flux and further we can use this flux to determine the local density of three-branes at the singularity.

To begin, we consider the superstar solution with one charge excited, i.e., $q_2 = q_3 = 0$. This configuration should correspond to a collection of giant gravitons moving along ϕ_1 with a certain distribution of sizes (specified by θ_1). As described above, we measure the density of giant gravitons sitting near a certain θ_1 by integrating $F^{(5)}$ over the appropriate surface. For an individual giant graviton, we could enclose a point on the brane at θ_1 with a small five-sphere in the r, θ_1, and ϕ_1 directions, as well as being spanned by the α_i. As ϕ_1 remains a Killing coordinate of the supergravity solution, the giant gravitons must be smeared along this direction, as is appropriate for a momentum state. Therefore we simply integrate over ϕ_1, independent of r and θ_1. We also integrate over θ_1 with fixed r, which is actually appropriate as we have a distribution of giant gravitons along the θ_1 direction, rather than a single brane at a fixed value of θ_1. From eq. (2.9), we read off the relevant five-form component as

$$F^{(5)}_{\theta_1\phi_1\alpha_1\alpha_2\alpha_3} = 2L^2 q_1 \sin\theta_1 \cos\theta_1 \sin^2\alpha_1 \sin\alpha_2\,. \tag{3.5}$$

Integrating over the relevant coordinates would then give a result proportional to n_1, the total number of D3-brane spheres distributed along the θ_1 direction — with our present conventions the result is $16\pi G_{10} T_3 n_1$. However, if we regard the surface as being very close to the sources (i.e., the radial distance is much smaller than the scale over which the density varies), we can find the density of branes along θ_1 by simply dropping the integral along this direction. The density of D3-branes at a certain θ_1 is then given by

$$\frac{dn_1}{d\theta_1} = \frac{N}{4\pi^3 L^4} \int F^{(5)}_{\theta_1\phi_1\alpha_1\alpha_2\alpha_3} d\phi_1 d^3\alpha = N \frac{q_1}{L^2} \sin 2\theta_1 \,. \tag{3.6}$$

The total number of giant gravitons is therefore

$$n_1 = \int_0^{\pi/2} d\theta_1 \, \frac{dn_1}{d\theta_1} = N \frac{q_1}{L^2} \,. \tag{3.7}$$

These calculations provide a confirmation of our conjecture as they indicate that the source correctly excites the five-form flux corresponding to a distribution of spherical D3-branes.

Now we further validate this result as follows: the test brane calculations showed that a giant graviton with 'radius' given by θ_1 has an angular momentum $P_{\phi_1} = N \sin^2\theta_1$. Applying this result to our source conjecture, the total angular momentum in this configuration would be given by

$$P_{\phi_1} = \int_0^{\pi/2} d\theta_1 \, \frac{dn_1}{d\theta_1} N \sin^2\theta_1 = \frac{N^2}{2} \frac{q_1}{L^2} \,, \tag{3.8}$$

which precisely coincides with the total internal momentum of the superstar! This is most easily seen using the five-dimensional BPS relation (3.2) which fixes the total mass of the superstar in terms of the internal momentum to be

$$E = \frac{P_{\phi_1}}{L} = \frac{N^2}{2} \frac{q_1}{L^3} \,. \tag{3.9}$$

To relate this result to the mass (2.6) of the superstar solution (with $q_2 = q_3 = \mu = 0$), we use the relations given in eqs. (3.3) and (3.4). We then find that eq. (2.6) may be rewritten as

$$M = \frac{N^2}{2} \frac{q_1}{L^3} \,, \tag{3.10}$$

in exact agreement with the above! Hence we have an interesting cross-check on our proposal that the source in the superstar corresponds to a distribution of giant gravitons.

We can easily extend these calculations to the general superstar solution with three non-zero charges. In this case, we think of the configuration as three distinct collections of giant gravitons, separately moving along one of the ϕ_i $(i = 1, 2, 3)$. We can think of the S^5 as embedded in $\mathbb{R}^6$ coordinates $x^{1,\dots,6}$,

$$x^{2i-1} = L\mu_i \cos\phi_i \,, \qquad x^{2i} = L\mu_i \sin\phi_i \,. \tag{3.11}$$

A giant graviton moving along ϕ_i therefore has a radius $\rho_i = L\sqrt{1-\mu_i^2}$. In analogy with above, calculating the density of gravitons of a certain radius as above then involves the $F^{(5)}$ component

$$F^{(5)}_{\rho_i\phi_i\alpha_1\alpha_2\alpha_3} = \frac{d\mu_i^2}{d\rho_i} F^{(5)}_{\mu_i^2\phi_i\alpha_1\alpha_2\alpha_3} = 2\rho_i q_i \,\sin^2\alpha_1\,\sin\alpha_2 \,. \tag{3.12}$$

The corresponding density of giant gravitons for each direction is

$$\frac{dn_i}{d\rho_i} = 2N\frac{q_i}{L^4}\rho_i \,, \tag{3.13}$$

which agrees with eq. (3.6) for $i = 1$. With this parameterization, a giant graviton of radius ρ_i carries angular momentum $P_{\phi_i} = N\rho_i^2/L^2$, in analogy with eq. (3.1). With this result and our graviton distributions (3.13), we find that the total angular momentum carried by each set of giant gravitons is

$$P_{\phi_i} = \frac{N^2}{2}\frac{q_i}{L^2} \,. \tag{3.14}$$

Just as above, these results correspond to the total angular momenta calculated for the superstar solution, and we have complete agreement between the BPS mass of the superstar (2.6) and the total energy of the giant gravitons, $E = \sum P_{\phi_i}/L$.

4. Dual giant probes

Recently string theorists have seen that certain supergravity singularities are resolved by the expansion of branes, e.g., [2, 4]. In the present context, as well as the giant gravitons expanding on the five-sphere, one has dual giant gravitons which expand in the anti-de Sitter geometry [10, 11, 12]. Hence one might be tempted to conjecture that these configurations play a role in resolving the superstar singularity, which appears at the origin of the AdS space. To investigate this possibility, we examined the behavior of dual giant graviton probes in the superstar background. Here we present a brief account of our calculations, however, we found no evidence favoring such an expansion in the AdS_5 directions.

The dual giant gravitons are spherical D3-branes, spanning the S^3 in the AdS_5 space at a constant value of r. We let the brane orbit on the S^5 at constant angles θ_1 and θ_2 with fixed values of the angular momenta, P_{ϕ_i}, conjugate to the angles ϕ_i.

The appropriate D3-brane probe action in the superstar background is given by

$$\begin{aligned} S_3 &= -T_3 \int dt d^3\alpha \left[\sqrt{-g} + B^{(4)}_{t\alpha_1\alpha_2\alpha_3} + \sum_i \dot\phi_i B^{(4)}_{\phi_i\alpha_1\alpha_2\alpha_3} \right] \\ &= \frac{N}{L^4} \int d^4\sigma \left[-r^3\Delta \sqrt{\frac{f}{H_1H_2H_3} - \frac{1}{\Delta}\sum H_i\mu_i^2 \left(L\dot\phi_i + \frac{1}{H_i} - 1 \right)^2} + \right. \\ &\qquad \left. + \frac{r^4}{L}\Delta + L\sum q_i\mu_i^2(L\dot\phi_i - 1) \right] . \end{aligned} \tag{4.1}$$

We fix the value of $p_i \equiv P_{\phi_i}/N$ and after some straightforward calculations, we find the Hamiltonian

$$\begin{aligned} \mathcal{H}(r,\mu_i) &= \frac{N}{L} \left[\sqrt{\sum \frac{\mu_i^2}{H_i} \left(1 + H_1H_2H_3\frac{r^2}{L^2} \right)} \times \right. \\ &\quad \times \sqrt{\left[\sum \frac{1}{H_j\mu_j^2} \left(p_j - \frac{q_j\mu_j^2}{L^2} \right)^2 + H_1H_2H_3\frac{r^6}{L^6}\sum \frac{\mu_j^2}{H_j} \right]} + \\ &\quad \left. + \sum \left(1 - \frac{1}{H_i} \right) \left(p_i - \frac{q_i\mu_i^2}{L^2} \right) - H_1H_2H_3\frac{r^4}{L^4}\sum\frac{\mu_i^2}{H_i} + \sum\frac{q_i\mu_i^2}{L^2} \right] . \end{aligned} \tag{4.2}$$

We want to minimize this energy with respect to r, θ_1 and θ_2. We find that there is a non-trivial minimum at a finite radius,

$$\frac{r^2}{L^2} = \sum_{i=1}^{3} \left(p_i - \frac{q_i\mu_i^2}{L^2} \right) , \tag{4.3}$$

when θ_1 and θ_2 are such that the μ_i solves the equations

$$\mu_i^2 = \frac{p_i - q_i\mu_i^2/L^2}{\sum \left(p_j - q_j\mu_j^2/L^2 \right)} . \tag{4.4}$$

These equations also imply that $L\dot\phi_i = 1$, which is a familiar equation from the giant graviton analysis. The energy at this minimum is simply

$$\mathcal{H}_{\min} = \frac{N}{L}\sum p_i = \frac{1}{L}\sum P_{\phi_i} \tag{4.5}$$

in accord with the BPS bound. In most regions of the parameter space there is also a degenerate minimum at $r^2 = -q_3$ or $\rho = 0$. The only exceptions to this are when $q_2 = q_3 = 0$ and p_2 or p_3 is non-zero (then $\mathcal{H}(r=0) > p_1$), or when $q_2 \neq 0, q_3 = 0, p_3 \neq 0$ (then $\mathcal{H}(r=0)$ is infinite) — we will return to discussing these exceptional cases in the next section.

Let us denote the angular momentum (divided by N) associated with the background charges as $\widetilde{p}_i$, such that according to eq. (3.14),

$$\widetilde{p}_i \equiv \frac{N}{2}\frac{q_i}{L^2}\,. \tag{4.6}$$

We see that in analogy with dual giant gravitons in the $\mathrm{AdS}_5 \times S^5$ vacuum, there are (in most cases) two solutions: one zero-size graviton, and one puffed-up dual giant. This is consistent with our interpretation in the previous section that the source should be simply giant gravitons distributed on the five-sphere. That is, because of the BPS properties of this system, we are free to add additional angular momentum by introducing dual giant gravitons. However, our test brane analysis shows that there is no reason that the additional momentum must be carried by probes must expanded in the AdS_5 directions. In particular, as all of the p_i become small, the probe collapses onto the singularity. Hence, our brane probe analysis provides no compelling evidence for an expansion of the superstar source in the AdS_5 directions.

Let us explicitly write out the radius (in the r coordinates) of the dual giant graviton in a few simple cases:

If only one of the probe angular momenta, say p_1, is non-zero, we get from eq. (4.4) that $\mu_1 = 1$, and

$$\frac{r^2}{L^2} = p_1 - \frac{2}{N}\widetilde{p}_1\,, \tag{4.7}$$

very similar to the results for a pure $\mathrm{AdS}_5 \times S^5$ background with $\widetilde{p}_i = 0$, i.e., $r^2/L^2 = p_1$ in the latter case [10, 11]. There is an interesting correction (which is small in the large N limit) due to the background. One also finds that increasing $\widetilde{p}_1$ lowers the potential barrier (in the Hamiltonian) between the zero-size graviton and the corresponding dual giant.

Another example is when the three background charges are equal, $\widetilde{p}_1 = \widetilde{p}_2 = \widetilde{p}_3 = \widetilde{p}$. Then we find $\mu_i^2 = p_i/\sum p_j$, such that the radius is given by

$$\frac{r^2}{L^2} = \sum p_i - \frac{2}{N}\widetilde{p}\,. \tag{4.8}$$

This again is close to the corresponding case in pure $\mathrm{AdS}_5 \times S^5$, where one finds $\mu_i^2 = p_i/\sum p_j$ as well as the radius $r^2/L^2 = \sum p_i$. In fact, in terms of the ρ coordinate (2.11), eq. (4.8) becomes precisely $\rho^2/L^2 = \sum p_i$.

We have also considered other cases, but the general analysis for the most generic situation is very complicated and yields no interesting insights.

The center of mass of the dual giant can be thought of as sitting at the (singular) center of the geometry where $r^2 = -q_3$ ($\rho = 0$), so we might expect this to be moving along a null trajectory. We can check this by evaluating ds^2 for $L\dot{\phi}_i = 1$ and $r^2 = -q_3$, and we find

$$ds^2\big|_{\mathrm{com}} = -\frac{\mu_3}{L^2}\sqrt{(q_1 - q_3)(q_2 - q_3)}\,. \tag{4.9}$$

We see that, curiously, this is not on a null trajectory in the generic cases. However, in many special cases we do get $ds^2 = 0$, when $p_3 = 0$ (which gives $\mu_3 = 0$) or when $q_2 = q_3$.

5. Discussion

We studied the properties of various charged BPS supergravity solutions of the type-IIB theory in asymptotically $\mathrm{AdS}_5 \times S^5$ spacetime. While these spacetimes contain naked singularities, it is natural to interpret such singularities in terms of an external source [17]. We argued that type-IIB string theory provides such a source for these solutions in the form of an ensemble of giant gravitons. These giant gravitons were detected by the dipole field which they excited in the RR five-form near the singularity. Thus the naked singularities appearing here are no worse than, e.g., those in the backgrounds representing planar D-branes. It is interesting to speculate that on some microscopic level, the solutions should be completely nonsingular as the sources are essentially expanded D3-branes which have a nonsingular core [26].

With the advent of the AdS/CFT correspondence [7, 8], there has been great interest in generating and studying asymptotically AdS solutions. However, unfortunately, one finds that many of these solutions contain naked singularities — see, e.g., [27] — and so the question of understanding when these singularities are physically acceptable has received some attention [2, 28, 29, 30, 31]. Given that the superstars are singular solutions which we are confident have a physical interpretation in string theory, they provide an interesting test of the various criteria which have been proposed. The proposal of ref. [29] refers to the behavior of g_{tt} near the singularity, where ∂_t is the Killing vector generating time translations in the dual field theory. While naively the superstar satisfies their criteria, it seems that their proposal should be refined to account for the present solutions. An interesting characteristic of the superstars is that in all cases, there is an ergoregion in the vicinity of the singularity, i.e., a region in which $g_{tt} > 0$! Further, generically $g_{tt} \to +\infty$ as one approaches the singularity and so the physical reasoning of ref. [29] should be reassessed. The criterion set forward in ref. [30] requires a Poincaré invariant background and so cannot be applied to the present solutions. However, ref. [30] also presents an interesting physical discussion to argue that singularities appearing in the zero temperature limit of a family of black hole solutions should be acceptable. From this point of view, the superstar would be deemed to contain a physically acceptable singularity.

As an interesting confirmation of giant gravitons as the source in the superstars, we combined the distribution (3.6) with the angular momentum-angle relation (3.1) derived with test-branes and we found that the total angular momentum precisely matches the total angular momentum of the solution. While an intriguing result, it is not immediately obvious why the (P_{ϕ_1}, θ_1) relation derived from a test-brane analysis should apply in a regime where the branes are generating strong gravitational and

RR fields. For the solutions with two (or three) charges, this reasoning seems to be consistent with a description of two (or three) independent sets of giant gravitons separately orbiting the five-sphere.

Within the test-brane analysis, one finds for a fixed P_{ϕ_1} that there are three candidate single brane configurations: a giant graviton expanded on the five-sphere, a dual giant expanded in the anti-de Sitter geometry and a point-like graviton. Interpretations within the dual CFT have been suggested for the first two of these configurations. Giant gravitons are argued to correspond to certain subdeterminant operators [9], while the dual giants seem to correspond to semiclassical coherent states [11]. The role of the zero-size configuration remains open, although some evidence has been given that these states do not exist in the CFT [11]. Further there is some question as to whether these configurations actually represent distinct states or dual descriptions of the same states, in analogy with the results of ref. [2]. A complementary possibility is that while distinct configurations, the underlying states may mix in the quantum theory [10, 32]. Hence we may ask if our present calculations can shed any light on these issues.

First our calculations indicate that all of the internal momentum of the superstar solutions is accounted for by giant gravitons. In particular then, there is no evidence of any zero-size gravitons in the ensemble. While not direct evidence that such states do not exist, this result is certainly consistent with this hypothesis.

We also considered probe calculations. Since the five-sphere at the center of the AdS_5 space is singular, we did not use giant gravitons, but rather dual giant probes. Here the original motivation had been to investigate whether brane expansion in the AdS directions played a role in resolving the singularity, in analogy to the enhançon effect [4]. However, in general there were two stable configurations: an expanded dual giant and a point-like state, where the probe collapsed to the origin. Thus there was no compelling evidence that the physical source should expand into the AdS space. Of course, we should remark that even if the sources are said to be point-like in the AdS space, we expect that in the full string theory their true extent is the size of the string-scale. For the present analysis, of course, this size is negligible compared to the macroscopic distance scales which appear in the background supergravity solutions.

However, there were a couple of interesting exceptions to the general result described above. For example, if $q_1 \neq 0$ but $q_2 = q_3 = 0$, a dual giant probe with $p_2 \neq 0$ had a single supersymmetric minimum at $r^2/L^2 = p_2$ with $E_{\rm BPS} = N\,p_2/L$. The Hamiltonian also has a local minimum at $r = 0$ but $E = N\,p_2/L\,\sqrt{1 + 2\tilde{p}_1/N} > E_{\rm BPS}$. Hence it seems that to add momentum along ϕ_2 to the original configuration the resulting BPS configuration must be extended in the AdS space.

Now it is interesting to note then that in the BPS background with both $q_1 \neq 0 \neq q_2$ (and $q_3 = 0$), if we examine the volume of the three-sphere in the AdS space as $r \to 0$ that the limiting volume remains finite, i.e., $V_{\Omega_3} \to 2\pi^2(q_1 q_2)^{3/4}|\mu_3|^{3/2}$. This result seems consistent with the previous observation in that the source in the full solution of the nonlinear supergravity equations also remains extended in the AdS space.

There is a similar result for dual probes with $p_3 \neq 0$ in the background with $q_1 \neq 0 \neq q_2$ and $q_3 = 0$. Here again the dual probes have a single supersymmetric minimum with $r^2/L^2 > 0$. Similarly the generic background solution with all three charges excited has the property that the volume of the three-sphere in the AdS space has a finite limit as $\rho \to 0$.

We would interpret these results as indicating that there is a mixing between the giant gravitons and their dual giant cousins, at least in configurations with more than one charge excited. No such expansion occurs for the simplest case where only q_1 is nonvanishing. That is, $V_{\Omega_3} \to 0$ as $r \to 0$ in this supergravity solution, and the dual giants carrying P_{ϕ_1} in this background collapse to zero size as the momentum gets small. Hence this mixing is not of the type envisioned in refs. [10, 32]. Rather it results from the interaction of different types of giant gravitons carrying internal momentum along orthogonal directions.

We should point out that from a ten-dimensional point of view, our superstar solutions are far from generic. They all have the distinguishing feature that they have been lifted from solutions of the five-dimensional gauged supergravity theory. Hence they only excite a very specific (and limited) set of the modes in the full ten-dimensional supergravity theory. Since the giant gravitons are all BPS and preserve a common set of supersymmetries independent of their internal momenta [10], we should expect that arbitrary distributions of giant gravitons can be arranged on the five-sphere. It might be interesting to examine such distributions to get a better understanding of the physics of giant gravitons. In particular, it would be interesting to find the background solution corresponding to a single giant graviton or rather a collection of giant gravitons, each of which carries the same P_{ϕ_1} and hence expands to the same size on the five-sphere.

Another property of the present solutions is that they are smeared in the momentum directions, i.e., they are invariant under rotations in ϕ_i. This raises two interesting questions which deserve further investigation. First, one might argue that this smearing is appropriate for a momentum eigenstate. On the other hand, most of the analysis of giant gravitons regards them as classical configurations which solve the test brane equations of motion. From this point of view, they are inherently 'localized' objects which follow a specific trajectory on the five-sphere. It would be interesting to understand whether there is a delocalization effect in the full quantum theory, i.e., the dual field theory, analogous to that studied in ref. [33]. It could be that the delocalized character of the supergravity solutions reflects an inherent property of the giant graviton states, rather simply indicating a need for better solution generating technology.

The smeared nature of the supergravity solutions is also interesting since it seems this property must be related to the 'rapid' fall-off of the fields in the asymptotic

$\mathrm{AdS}_5 \times S^5$ region. Ref. [34] showed that the disturbance generated by a source that is 'point-like' or spherical from a ten-dimensional point of view is exponentially large in the asymptotic region. The superstars clearly do not exhibit this behavior, which must be because of their relatively smooth character on the five-sphere. It would be interesting to know if such refocussing of the higher multipole moments occurs for an individual giant graviton. One approach to address this question would be study the gravitational backreaction for a single giant graviton treated as a classical test brane. A short calculation shows that the backreaction for an individual giant graviton is small. That is, the gravitational length scale introduced would be given by $r_g^2/L^2 \simeq P_{\phi_i}/N^2 \leq 1/N$. Hence we expect that this problem could be addressed within the context of linearized perturbations of the supergravity fields in the $\mathrm{AdS}_5 \times S^5$ background. As the analysis of ref. [34] focussed on static sources while a giant graviton is a dynamical configuration, the final result is not clear.

Studying the non-BPS solutions, i.e., those with $\mu \neq 0$, would also be of interest. For the single charge case, as soon as the nonextremality parameter is nonvanishing, an event horizon and the associated large entropy appears immediately. In the case of more than one charge, there is a gap between the BPS configuration and the black hole solutions. Presumably the intervening singular but non-BPS solutions are still physical (or perhaps only for a discrete spectrum) in analogy with the AdS_3 case — see, e.g., [35]. In terms of the giant gravitons, one might think these solutions are produced by adding new pairs of giant gravitons moving in the opposite directions on the S^5. This will increase the energy but not the internal angular momentum, and so supersymmetry will be broken. Since the RR five-form gauge-field doesn't change for non-zero μ, we find the same distribution of D3-branes as before. This can be understood from the fact that D3-branes moving in the 'wrong' direction on the S^5 will contribute negative D3-brane charge.

The full story for the nonextremal configurations, and in particular the black holes, is further complicated because it is known that these solutions suffer from certain instabilities. This was first suggested by the thermodynamic analysis of the charged black holes [18, 24, 25]. Further, a fluctuation analysis (for the analogous AdS_4 solutions) confirmed the existence of a classical perturbative instability [36]. It would be interesting to understand if giant gravitons or the dual giants play a role in the evolution of these solutions once the instability sets in. Probe calculations may provide some insight into this question.

An obvious extension of the present work is to study the analogous superstar solutions in eleven-dimensional supergravity. In this case, giant gravitons appear as expanding M5- or M2-branes in the $\mathrm{AdS}_4 \times S^7$ and $\mathrm{AdS}_7 \times S^4$ backgrounds [3, 10, 11]. Supergravity solutions describing charged black holes have been constructed in the reduced supergravities in four dimensions [37] and in seven dimensions [24]. Further the lift of these solutions to the full eleven-dimensional theory has been provided in ref. [19]. Repeating the analysis of sections 3 and 4 yields very similar results in

JHEP11(2001)009

those cases [38], and so the singularities in the corresponding superstars are again interpreted as being generated by ensembles of giant gravitons.

Acknowledgments

We thank Nick Evans, Clifford Johnson, Don Marolf, Simon Ross and Matt Strassler for useful discussions. RCM would also like to thank the Aspen Center for Physics for hospitality during various stages of this work. This research was supported by NSERC of Canada and Fonds FCAR du Québec.

References

[1] R.C. Myers, *Dielectric-branes*, *J. High Energy Phys.* **12** (1999) 022 [hep-th/9910053].

[2] J. Polchinski and M.J. Strassler, *The string dual of a confining four-dimensional gauge theory*, hep-th/0003136.

[3] J. McGreevy, L. Susskind and N. Toumbas, *Invasion of the giant gravitons from anti-de Sitter space*, *J. High Energy Phys.* **06** (2000) 008 [hep-th/0003075].

[4] C.V. Johnson, A.W. Peet and J. Polchinski, *Gauge theory and the excision of repulson singularities*, *Phys. Rev.* **D 61** (2000) 086001 [hep-th/9911161].

[5] J. Maldacena and A. Strominger, *AdS_3 black holes and a stringy exclusion principle*, *J. High Energy Phys.* **12** (1998) 005 [hep-th/9804085];
A. Jevicki and S. Ramgoolam, *Non-commutative gravity from the AdS/CFT correspondence*, *J. High Energy Phys.* **04** (1999) 032 [hep-th/9902059];
P.-M. Ho, S. Ramgoolam and R. Tatar, *Quantum spacetimes and finite n effects in 4d super Yang-Mills theories*, *Nucl. Phys.* **B 573** (2000) 364 [hep-th/9907145];
S.S. Gubser, *Can the effective string see higher partial waves?*, *Phys. Rev.* **D 56** (1997) 4984 [hep-th/9704195].

[6] H.J. Kim, L.J. Romans and P. van Nieuwenhuizen, *The mass spectrum of chiral $N=2$ $D=10$ supergravity on S^5*, *Phys. Rev.* **D 32** (1985) 389;
M. Gunaydin and N. Marcus, *The spectrum of the S^5 compactification of the chiral $N=2$, $D=10$ supergravity and the unitary supermultiplets of* $\mathrm{U}(2,2/4)$, *Class. and Quant. Grav.* **2** (1985) L11.

[7] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[8] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rep.* **323** (2000) 183 [hep-th/9905111].

[9] V. Balasubramanian, M. Berkooz, A. Naqvi and M.J. Strassler, *Giant gravitons in conformal field theory*, hep-th/0107119.

[10] M.T. Grisaru, R.C. Myers and O. Tafjord, *SUSY and Goliath*, *J. High Energy Phys.* **08** (2000) 040 [hep-th/0008015].

[11] A. Hashimoto, S. Hirano and N. Itzhaki, *Large branes in AdS and their field theory dual*, *J. High Energy Phys.* **08** (2000) 051 [hep-th/0008016].

[12] E. Bergshoeff, M.J. Duff, C.N. Pope and E. Sezgin, *Compactifications of the eleven-dimensional supermembrane*, *Phys. Lett.* **B 224** (1989) 71.

[13] K. Behrndt, A.H. Chamseddine and W.A. Sabra, *BPS black holes in $N = 2$ five dimensional AdS supergravity*, *Phys. Lett.* **B 442** (1998) 97 [hep-th/9807187].

[14] K. Behrndt, M. Cvetič and W.A. Sabra, *Non-extreme black holes of five dimensional $N = 2$ AdS supergravity*, *Nucl. Phys.* **B 553** (1999) 317 [hep-th/9810227].

[15] L.J. Romans, *Supersymmetric, cold and lukewarm black holes in cosmological Einstein-Maxwell theory*, *Nucl. Phys.* **B 383** (1992) 395 [hep-th/9203018];
L.A.J. London, *Arbitrary dimensional cosmological multi-black holes*, *Nucl. Phys.* **B 434** (1995) 709.

[16] G.T. Horowitz and R.C. Myers, *The value of singularities*, *Gen. Rel. Grav.* **27** (1995) 915 [gr-qc/9503062].

[17] C.V. Johnson, N. Kaloper, R.R. Khuri and R.C. Myers, *Is string theory a theory of strings?*, *Phys. Lett.* **B 368** (1996) 71 [hep-th/9509070].

[18] A. Chamblin, R. Emparan, C.V. Johnson and R.C. Myers, *Charged AdS black holes and catastrophic holography*, *Phys. Rev.* **D 60** (1999) 064018 [hep-th/9902170]; *Holography, thermodynamics and fluctuations of charged AdS black holes*, *Phys. Rev.* **D 60** (1999) 104026 [hep-th/9904197].

[19] M. Cvetič et al., *Embedding AdS black holes in ten and eleven dimensions*, *Nucl. Phys.* **B 558** (1999) 96 [hep-th/9903214].

[20] M.J. Duff, *TASI lectures on branes, black holes and anti-de Sitter space*, hep-th/9912164.

[21] J.H. Schwarz, *Spontaneous compactification of extended supergravity in ten dimensions*, *Physica* **A124** (1984) 543.

[22] H. Nastase, D. Vaman and P. van Nieuwenhuizen, *Consistent nonlinear KK reduction of 11d supergravity on $AdS_7 \times S^4$ and self-duality in odd dimensions*, *Phys. Lett.* **B 469** (1999) 96 [hep-th/9905075];

JHEP11(2001)009

H. Nastase, D. Vaman and P. van Nieuwenhuizen, *Consistency of the $AdS_7 \times S^4$ reduction and the origin of self-duality in odd dimensions*, *Nucl. Phys.* **B 581** (2000) 179 [hep-th/9911238];
H. Nastase and D. Vaman, *On the nonlinear KK reductions on spheres of supergravity theories*, *Nucl. Phys.* **B 583** (2000) 211 [hep-th/0002028];
H. Nastase and D. Vaman, *The AdS-CFT correspondence, consistent truncations and gauge invariance*, hep-th/0004123.

[23] M. Gunaydin, G. Sierra and P.K. Townsend, *Gauging the $D = 5$ maxwell-einstein supergravity theories: more on Jordan algebras*, *Nucl. Phys.* **B 253** (1985) 573.

[24] M. Cvetič and S.S. Gubser, *Phases of R-charged black holes, spinning branes and strongly coupled gauge theories*, *J. High Energy Phys.* **04** (1999) 024 [hep-th/9902195].

[25] M. Cvetič and S.S. Gubser, *Thermodynamic stability and phases of general spinning branes*, *J. High Energy Phys.* **07** (1999) 010 [hep-th/9903132].

[26] I.R. Klebanov, *World-volume approach to absorption by non-dilatonic branes*, *Nucl. Phys.* **B 496** (1997) 231 [hep-th/9702076].

[27] S.S. Gubser, *Dilaton-driven confinement*, hep-th/9902155;
N.R. Constable and R.C. Myers, *Exotic scalar states in the AdS/CFT correspondence*, *J. High Energy Phys.* **11** (1999) 020 [hep-th/9905081];
D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Continuous distributions of D3-branes and gauged supergravity*, *J. High Energy Phys.* **07** (2000) 038 [hep-th/9906194].

[28] N. Evans, C.V. Johnson and M. Petrini, *The enhancon and $N = 2$ gauge theory/gravity RG flows*, *J. High Energy Phys.* **10** (2000) 022 [hep-th/0008081];
A. Buchel, A.W. Peet and J. Polchinski, *Gauge dual and noncommutative extension of an $N = 2$ supergravity solution*, *Phys. Rev.* **D 63** (2001) 044009 [hep-th/0008076].

[29] J. Maldacena and C. Nunez, *Supergravity description of field theories on curved manifolds and a no go theorem*, *Int. J. Mod. Phys.* **A 16** (2001) 822 [hep-th/0007018].

[30] S.S. Gubser, *Curvature singularities: the good, the bad and the naked*, hep-th/0002160.

[31] H.D. Kim, *A criterion for admissible singularities in brane world*, *Phys. Rev.* **D 63** (2001) 124001 [hep-th/0012091].

[32] J. Lee, *Tunneling between the giant gravitons in $AdS_5 \times S^5$*, *Phys. Rev.* **D 64** (2001) 046012 [hep-th/0010191].

[33] D. Marolf and A. Peet, *Brane baldness vs. superselection sectors*, *Phys. Rev.* **D 60** (1999) 105007 [hep-th/9903213];
S. Surya and D. Marolf, *Localized branes and black holes*, *Phys. Rev.* **D 58** (1998) 124013 [hep-th/9805121].

[34] G.T. Horowitz and V.E. Hubeny, *CFT description of small objects in AdS*, *J. High Energy Phys.* **10** (2000) 027 [hep-th/0009051].

[35] V. Balasubramanian, J. de Boer, E. Keski-Vakkuri and S.F. Ross, *Supersymmetric conical defects: towards a string theoretic description of black hole formation*, *Phys. Rev.* **D 64** (2001) 064011 [hep-th/0011217];
J. de Boer, *Six-dimensional supergravity on $S^3 \times AdS_3$ and 2d conformal field theory*, *Nucl. Phys.* **B 548** (1999) 139 [hep-th/9806104].

[36] S.S. Gubser and I. Mitra, *Instability of charged black holes in anti-de Sitter space*, hep-th/0009126;
S.S. Gubser and I. Mitra, *The evolution of unstable black holes in anti-de Sitter space*, *J. High Energy Phys.* **08** (2001) 018 [hep-th/0011127].

[37] M.J. Duff and J.T. Liu, *Anti-de Sitter black holes in gauged $N = 8$ supergravity*, *Nucl. Phys.* **B 554** (1999) 237 [hep-th/9901149];
W.A. Sabra, *Anti-de Sitter BPS black holes in $N = 2$ gauged supergravity*, *Phys. Lett.* **B 458** (1999) 36 [hep-th/9903143].

[38] F. Leblond, D.C. Page and R.C. Myers, *Superstars and giant gravitons in M-theory*, in preparation.

JHEP11(2001)009